SIMULATION, OPTIMIZATION AND EVALUATION OF SYSTEMS OF TRAFFIC NETWORKS

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1. Introduction:

Since the end of World War II, the United States, as well as other parts of the world, have witnessed an unprecedented growth in economic well being. This, in turn, has produced an accelerating increase in the demands for transportation which is both a cause and a consequence of this well being. A "research revolution" has also been in process all during this time which has, in its own right, contributed to this growth and, of course, this same revolution has also provided opportunities for securing new and improved modes of transport as well as augmenting the ways in which the already existing transport technology can be utilized.

At this point it should perhaps be emphasized that this research revolution has encompassed more than only hardware development in its range of possibilities. Thus, for instance, the period since World War II has also witnessed a large amount of research and a concommitant increase in the development and use of new managerial tools. This includes uses of electronic computers and other aids for data processing and communication. It also includes developments in areas like the modelling of managerial problems which range from (1) raw (unguided) simulations of situational possibilities and extend to (2) highly sophisticated mathematical models which, in turn, have required the development of new mathematics for their efficient use.

It is to this latter area that the present paper is addressed. First, it is proposed to show here how certain types of mathematical (optimization) models may be developed and used when, for instance, a proposed simulation

might otherwise be too unwieldly or too complicated to execute. Second, it is proposed to show how these optimizations may be extended very naturally in order to answer still other questions that might be of interest. All of this will be done, however, in a loose, heuristic manner.

It might also be well to note that we are here utilizing a "systems approach" to designing a city street network. Thus, we are not here addressing the problem of new modes of transport *per se*--although undoubtedly a systems approach might also be in order for such purposes--but are concerned rather with the problem of design as it might be encountered when considering how to plan the improvement of an existing network of streets.

2. **Background:**

As a first step toward introducing the subject at issue we may consider how it was encountered at CATS (Chicago Area Transportation Study) under the directorship of Dr. J. Douglas Carroll a few years ago. At an early date in the CATS program, certain surveys had been undertaken to determine points of traffic origin and destination. Thus, for instance, one approach involved the use of "cordon lines" where all traffic was halted and drivers were interrogated with reference to (a) where their trip had originated and (b) where their destinations were located.[1/] A house-to-house survey was also conducted to elicit similar

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1/ The drivers were also queried as to destinations that had already been reached.

information. That is, a sample of households were interviewed for the purpose of determining their normal transport destinations and requirements on typical days.

The resulting data were then aggregated into 100 major points of origin and an equal number of destinations. It was then proposed to secure a better understanding of traffic behavior by utilizing these data as part of a traffic simulation study. To understand what is involved for such a study it is, in principle, merely necessary to imagine that these data are all assembled at their respective points of origin on a map of the street network of the city of Chicago. Given the intended destinations one might then locate a variety of possible routes by means of which the indicated traffic loads could be distributed in order that the resulting vehicular traffic would reach the required destinations.

Of course, any such assignment of traffic to routes (between all origins and all destinations) would have to consider the resulting volumes on every link of the existing city-street network if only because this would affect the travel time requirements.[1/] This, in turn, would require the consideration of alternate routing possibilities along with their related time-volume consequences, and so on. Even at this very simple level the problem of effecting such assignments was so huge that hand-simulations were out of the question. Indeed, the size of the problem was

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1/ Observations on these time-volume relationships were also obtained by CATS.

further complicated by the nonlinear character of the time-volume response and two-way traffic flow possibilities so that even the attempt to use an electronic computer for these purposes was also frustrated.

Attention was therefore turned to some of the possibilities that might be offered by means of some of the newer developments in optimization mathematics. Note, however, that the initial objective was only to secure a simulation that would utilize these origin-destination data in a way that would effect traffic routings that corresponded to actually observed patterns of traffic. That is, the effectuation of such a routing was to be considered as a validation test of the simulation model and its associated optimization principle. However, as we shall see, it was possible finally to synthesize a model which did a good deal more. For instance, it was possible to utilize the model so that its underlying theory could also be brought directly to bear on the problem of redesign so that this part of the study could be coordinated with the simulations that had previously been perceived as only a separate preliminary. Indeed, such further possibilities are not even now wholly exhausted since, by additional research, it should be possible to obtain overall cost/benefit measures of a systems variety for the various redesign possibilities that might be considered. By still further research, it should also be possible to effect additional extensions whereby zoning and street redesign might be jointly and simultaneously considered. Finally, this would open a way for exploring alternatives to the present relatively

static modes of zoning that are now almost universally employed by American cities. Thus, in particular, one might then envision a zoning code which includes a dynamic conditional approach, with explicitly stated rules for rezoning, as an alternative to present, statically fixed, approaches which often provide incentives for affected property owners to deviate from the standards that supposedly governed when an original zoning was imposed.

3. A Linear Programming Model of Network Type:

Among the (largely) post World War II developments that suggested themselves for consideration in such a systems approach there were two that occupied a place of prominence by virtue of their orientation toward large systems with numerous interactions. These two were (a) linear programming and (b) the theory of n-person games. Actually, as we shall see, it was found desirable to amalgamate both of these approaches in order to join the conceptual power of the theory of the games with the computational power of linear programming. Even this accomplishment was not enough, however, and so, by virtue of still further research, certain extensions were finally effected that made it possible, finally, to obtain the kinds of results that were wanted.

Perhaps the easiest way to approach the indicated developments is via a linear programming model of "network" (or incidence) type. Therefore consider a network of this kind by reference to the particular one depicted in Figure 1 below. This Figure may be thought of as a network diagram in which the branches (or links) represent streets and the nodes represent possible points of

origin or destination for traffic which is to flow over this system.[1/] Thus, in this interpretation, the heavy inward-pointing arrow at node (1) is intended to symbolize that ten vehicles are to be entered into the system at this point. The heavy, outward-pointing arrows at nodes (5), (7), and (9) are intended to mean that quantities of two, five, and three vehicles have their destinations, respectively, at these nodes.

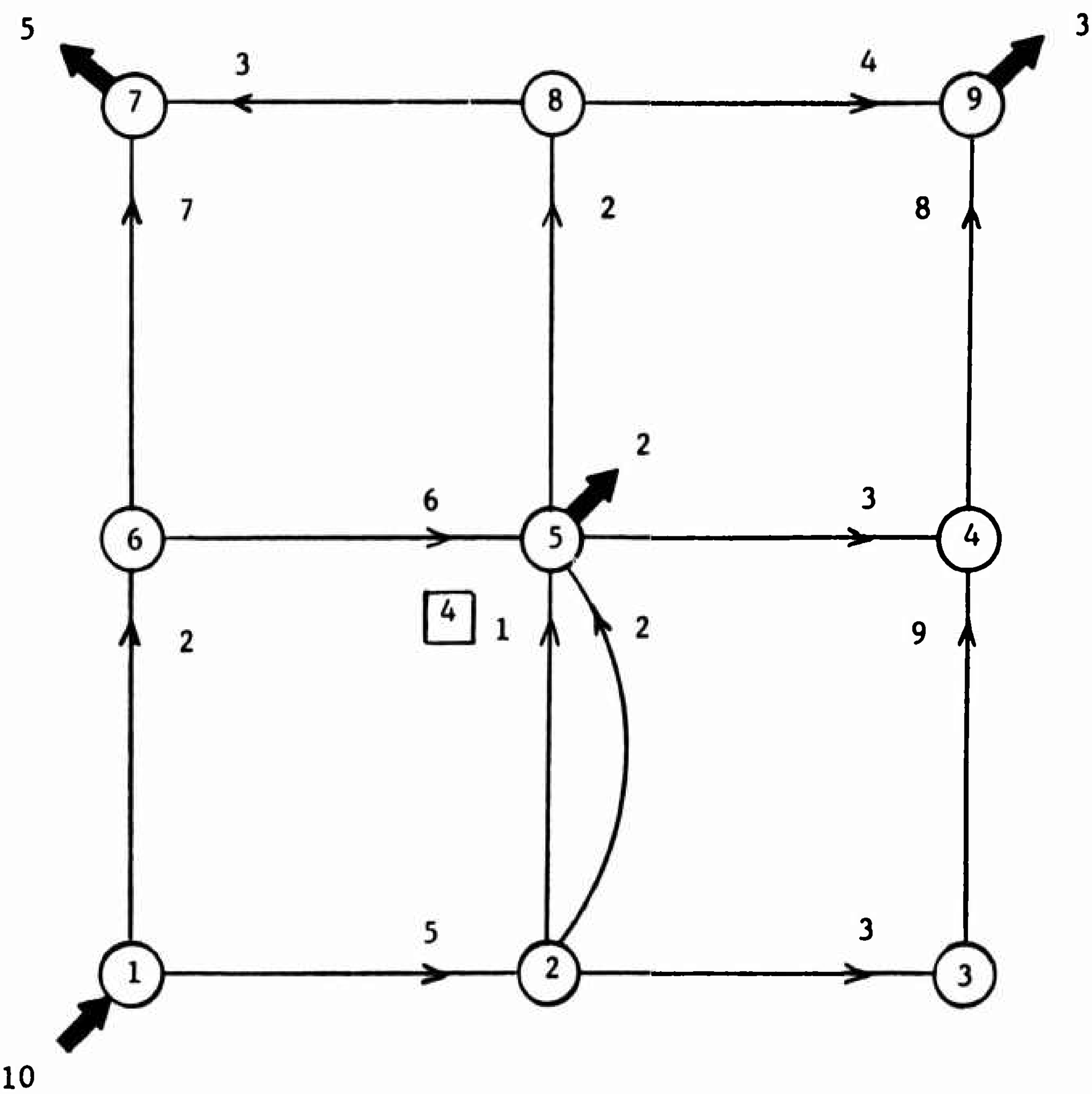

FIGURE 1 Single-source network copy

1/ Actually this network had previously been used as a prototype for analyses of the hydraulic behavior that might be encountered in designing a fire-water safety system in a refinery. See Chapter XVII in A. Charnes and W. W. Cooper, op. cit.

For our immediate purposes, we may imagine that these data are being used by a truck despatcher whose objective is to schedule the routings for these ten trucks in a way that will minimize the total number of vehicle hours required to fulfill their destination requirements. For this, of course, he will need the transit times likely to be experienced on the routes available to him.[1/] These are supplied by the numbers positioned alongside the arrowheads. Thus, for instance, if the despatcher were to route two vehicles via the branches 1-2 and 2-5, then he would expect a result in vehicle hours in the amounts 2 x 5 (from 1-2) plus 2 x 1 (from 2-5) for a total of twelve vehicle hours.

The two links between nodes (2) and (5) are intended to reflect a nonlinearity in that a transit time of 1 hour on this link is obtainable only when four or less vehicles are assigned to it. (See the $\boxed{4}$ opposite the straight-line link connecting nodes (2) and (5).) If more than four vehicles are assigned to this link, then the transit times will be increased as a function of the number of vehicles. To illustrate how this is to be handled, we may suppose, first, that five vehicles are assigned to this link. By the approximating fiction we are now using,[2/] the transit time is to be increased from 1 hour per vehicle to 1 1/5 hours per vehicle in accordance with the expression $(4 + \Delta 2)/(4 + \Delta)$ $= (4 + 1 \times 2)\ (4 + 1) = 6/5$. That is, for the purpose of computing total transit times, four vehicles are considered to experience a one-hour transit time and the fifth one is assigned a two-hour time of transit. Similarly

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1/ These may be obtained by suitable averaging devices or, if this is not satisfactory, and if the relevant probability distributions are available, recourse may be had to the extensions available from chance constraints or stochastic linear programming. See, e.g., A. Charnes, W. W. Cooper, J. K. DeVoe and D. Learner: "DEMON: Decision Mapping via Optimum Go-No Networks: A Model for Introducing New Products", Management Science (forthcoming).

2/ Other fictions may also be used.

if six vehicles are assigned to this route, the average time of transit will be $(4+\Delta 2)/(4+\Delta) = (4+2\times 2)/(4+2) = 8/6$, or 1 1/3 hours per vehicle, and so on, so that even though the model discriminates between the first four vehicles and the still further increments that might be made, the result is, nevertheless, to be interpreted as a secured average transit time that is applicable to every vehicle which utilizes this link.

For certain purposes it is convenient to have an algebraic depiction of the model that is geometrically portrayed in Figure 1. This can be done conveniently by reference to a tabular array (or matrix) which can be synthesized as shown in Table 1, below. The nodes, as numbered in Figure 1, are first entered in the stub and the corresponding link designations are then entered at the head of the columns in any convenient order. Thus, for instance, the link connecting nodes 1 and 2 is designated for the first column while the link connecting nodes 1 and 6 is assigned to the second column, and so on, until, finally, the link connecting nodes 8 and 9 is entered in the last column. The transit times associated with these links are next entered at the foot of these columns and the stipulated traffic influxes and effluxes are then entered in the right-hand column (labelled "stipulations") where an entry of 0 indicates that the node in this row is neither an origin nor a destination but is only an in-transit node across which traffic may flow provided it is never allowed to remain there.[1/]

1/ I.e., these are essentially the same as Kirchoff's node conservation law in electrical circuit theory.

We can now proceed to complete the indicated algebraic model as follows. Observe that the link connecting nodes 1 and 2 does not touch (i.e., is not incident on) any other node. This is reflected by assigning values of $\pm$ 1 in accordance with the fact that this link is pointed toward 2 and away from 1 and assigning a zero (or blank) to every other node contact possibility in this same column. This same convention yields a 1 opposite node 1 and a -1 opposite node 6 in the column for link 1 - 6 in Table 1. Continuing in this fashion the other columns are completed in an analogous fashion with the result that each column contains only the two non-zero numbers: a plus one for the node on which the tail of its arrow is incident and a minus one for the node on which the head of its arrow is incident.

Links / Nodes	Variables													Stipulations
	q_1	q_2	q_3	q_4	q_5	q_6	q_7	q_8	q_9	q_{10}	q_{11}	q_{12}	q_{13}	
	1-2	1-6	2-3	2-5	2-5	3-4	5-4	4-9	6-5	5-8	6-7	8-7	8-9	
1	1	1												10
2	-1		1	1	1									0
3			-1			1								0
4						-1	-1	1						0
5				-1	-1		1		-1	1				-2
6		-1							1		1			0
7											-1	-1		-5
8										-1		1	1	0
9								-1					-1	-3
Transit times	5	2	3	1	2	9	3	8	6	2	7	3	4	

TABLE 1 INCIDENCE MATRIX FOR FIG. 1

Above each of the columns in Table 1 we now assign a variable $q_1, \dots, q_{16}$, the value of which corresponds to any assignable amount of vehicular traffic. Dropping these values into position we can then form the following array of equations as an algebraic correspond of all of the traffic assignment possibilities admitted by Figure 1:

$$
(1)\qquad
\begin{array}{rcccccccccccccc}
10 = & q_1 & + q_2 & & & & & & & & & & & \\
0 = & -q_1 & & + q_3 & + q_4 & + q_5 & & & & & & & & \\
0 = & & & - q_3 & & & + q_6 & & & & & & & \\
0 = & & & & & & - q_6 & - q_7 & + q_8 & & & & & \\
-2 = & & & & - q_4 & - q_5 & & + q_7 & & - q_9 & + q_{10} & & & \\
0 = & & - q_2 & & & & & & & + q_9 & & + q_{11} & & \\
-5 = & & & & & & & & & & & - q_{11} & - q_{12} & \\
0 = & & & & & & & & & & - q_{10} & & + q_{12} & + q_{13} \\
-3 = & & & & & & & & - q_8 & & & & & - q_{13}
\end{array}
$$

To see what has been accomplished thus far refer to the first equation, above, and note that q_1 is associated with link 1-2 while q_2 is associated with link 1-6. Thus

$$
(2)\qquad 10 = q_1 + q_2
$$

indicates that any assignment when summed over these two links must accommodate the 10 vehicles that are to enter the system at node 1. No other links, hence no other variables, are directly available for this assignment. See Figure 1 and Table 1.

These same variables also appear in euqations 2 and 6, as follows,

$$(3)\quad \begin{aligned} 0 &= -q_1 + q_3 + q_4 + q_5 \\ 0 &= -q_2 + q_9 + q_{11} \end{aligned}$$

so that if, say, q_1 and q_2 were each assigned values of 5 vehicles then, respectively, $q_3 + q_4 + q_5 = 5$ and also $q_9 + q_{11} = 5$ in order to satisfy the expressions (3).

It should be emphasized that negative as well as positive values are admitted for these variables. For instance, q_9 (which is associated with link 6-5) might have been assigned negative value so that, say, $q_9 = -2$. This would then be interpreted to mean that the flow was counter to the orientation of the arrow connecting these two links. I.e., this would mean that this flow of 2 vehicles was in the direction 6 to 5 whereas a value of $q_9 = 2$ would indicate a flow to 5 from 6.

The direction of flow on each link, as well as the numerical value of the variable associated with it is determined by the optimization. Here, as already noted, the objective of this hypothetical truck despatcher is to minimize the total time required to move his 10 trucks from their one origin (at node 1) to their destinations in the requisite numbers. Thus, to enable him to judge this total time we form the following expression by dropping each of the variables alongside their transit times as noted at the bottom of Table 1--viz.,

$$(4) \qquad \begin{aligned} &5q_1 + 2q_2 + 3q_3 + q_4 + 2q_5 + 9q_6 + 3q_7 + 8q_8 \\ &\qquad + 6q_9 + 2q_{10} + 7q_{11} + 3q_{12} + 4q_{13} \end{aligned}$$

The model is now almost complete except that it is necessary also to account for the nonlinearity which is encountered whenever more than 4 vehicles are assigned for travel between nodes 2 and 5. Part of what is wanted for this purpose has already been accomplished by assigning two columns to link 2-5 in the above tabular array. The first of these columns is associated with variable q_4 which has a 1-hour link-transit time. The second of these columns is associated with variable q_5 which has a 2-hour transit time. (Observe the values for the coefficients of these two variables as they appear in (4).) Thus, if we now write

$$(5) \qquad -4 \leq q_4 \leq 4$$

to mean that no more than 4 vehicles can be assigned as the value of this variable, we can then say that our model is complete and interpret it verbally to mean: "minimize total travel time as defined by (4), while restricting the values of the variables so that they satisfy all of the conditions represented by (1) and (5)."

This identifies the problem as one of linear programming and hence the computational routines and computer codes, etc., identified with that discipline may be used for its solution.[1/] Indeed something more may be done and, in fact, especially efficient methods were devised for solving

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1/ See *op. cit.* Chapter X for the way in which linear programming may be extended to handle nonlinearities such are involved here.

models of this type.[1/] In any event an optimum can be secured and displayed as in Table 2, below, where variables associated with non-utilized links are assigned a zero value.

Link Utilized	Variable	Assigned Value	Time per Vehicle	Total Vehicle Hours
1-2	q_1	5	5	25
1-6	q_2	5	2	10
2-3	q_3†	0	3	0
2-5*	q_4	4	6/5	6
2-5*	q_5	1		
5-4	q_7†	0	3	0
5-8	q_{10}	3	2	6
6-7	q_{11}	5	7	35
8-9	q_{13}	3	4	12
Total	8	xxx	xxx	94

*Nonlinear link. Since this is really only a single (combined) variable, the total number of links utilized is totaled to 8.

†Included at zero since variable is part of the basic solution.

1/ See the discussion in Chapter XVII, op. cit.

Direct substitution of these results in (1) will show that all of these expressions are satisfied. E.g., for the first expression in this set,

$$10 = q_1 + q_2 = 5 + 5 .$$

While for the fifth, seventh and ninth equations, respectively,

$$- 2 = -q_4 - q_5 + q_7 - q_9 + q_{10} = -4-1 + 0 - 0 + 3$$
$$- 5 = -q_{11} - q_{12} = -5 - 0$$
$$- 3 = -q_8 - q_{13} = - 0 - 3,$$

so that, in particular, the origin and destination requirements are all satisfied. Finally, substitution in (5) or, equivalently, the tally performed in the last column of Table 2 gives a minimum value of 94 vehicle hours which these routings yield.

3. Some Immediate Extensions:

As is perhaps evident, further extensions were needed in order to accommodate what was required for CATS. Before turning to these in the next section, however, it is of some value to examine some of the further possibilities that are at hand in this more simple case.

The underlying computational routines provide certain byproducts that are often of interest in their own right. These include so-called "sensitivity analyses" which explore potential program reactions to variations in the transit times. Thus, for instance, it may be desirable

to explore the consequences of increasing the transit time on the nonlinear link 2-5 and thereby establish a limit at which alternate routings will be worth undertaking. Other variations are also possible, of course, and these sensitivity analyses may also be extended to evaluating alterations in the requirements for origins and destinations as well as the introduction of new links, eradication of existing links and so on.

The techniques for effecting such analyses are all well known and readily applied by reference to standard modes of interpretation and application from linear programming theory.[1/] Hence we shall not discuss them here. Instead we shall indicate possible routes for further elaboration which are also pertinent for the more complex model that follows.

It might be noticed, for instance, that the flows for Figure 1 were all isotropic in that the same transit times were applied irrespective of the direction of flow for the assignment to any link. However, anisotropic flow possibilities may also be admitted along with other details as required in a variety of ways.

To illustrate what is involved and also to indicate still additional possibilities we have lifted node 5 and decomposed it to allow for variations in time when left- or right-hand turns are to be executed.[2/]

1/ See, e.g., A. Charnes and W. W. Cooper, op. cit. or G. B. Dantzig, Linear Programming and Extensions (Princeton, N. J.: Princeton University Press, 1963).

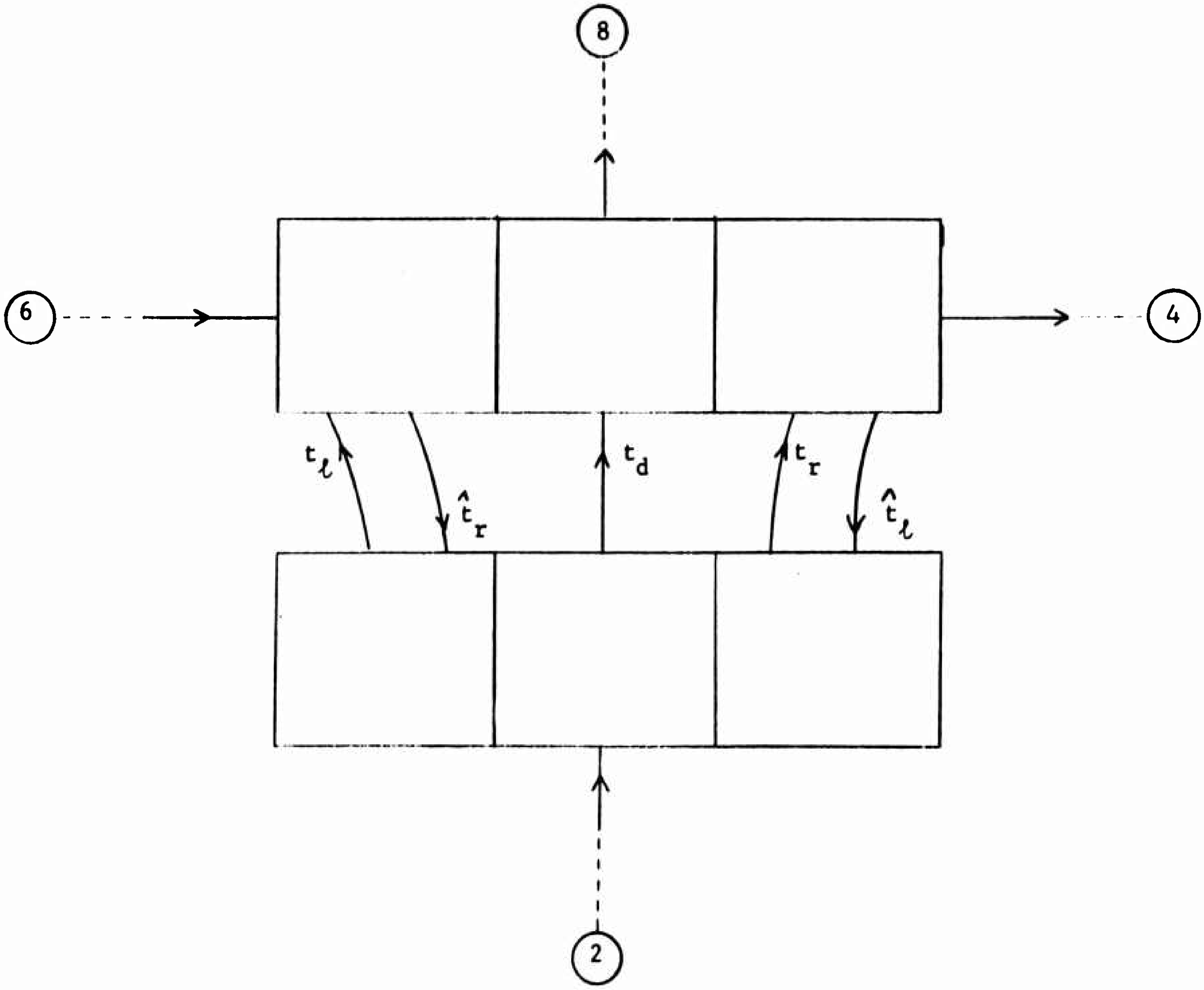
2/ This is only a variant of ideas contained in J. A. Wattleworth and P. W. Shuldiner, "The Inclusion of Left Turns in Traffic Prediction Models of the Charnes-Cooper Type," Proceedings of the ASCE Transportation Engineering Conference, October, 1962.

Such further detail might be pertinent when, for instance, a stop-light located at this node is timed to distinguish between these different types of turns--viz., t_ℓ represents time required (e.g., on the average) in effecting a left-hand turn when proceeding from node (2) to node (6) via node (5) while $\hat{t}_r$ represents the possibly different time required when proceeding from node (6) to node (2) via node (5) . 1/

1/ Such further detail may be added <u>after</u> (as well as before) effecting the initial allocations. Then after an initial optimum has been achieved the left-turn requirements actually achieved may then be submitted to a sensitivity analysis by augmenting the times on the link preceding the left-turns in order to study whether these augmentations have any implications for traffic reassignments.

Figure 2

Detail for Node 5
with
Left- and Right-Hand Turns



In this representation[1] the variables associated with the t_ℓ and t_r values are constrained to be non-negative, but this is not the case for the variable associated with t_d where flow is allowed in either direction so that the value of this variable may be either positive or negative as the optimization may require. Thus, as should be evident, any combination of isotropic and anisotropic flow possibilities may be accommodated link-by-link in the network. Also where one-way traffic only is permitted on any link an appropriate orientation of the arrow and a non-negativity requirement imposed on the associated variable produces what is wanted. If left hand turns are to be interdicted at any node then one assigns this t_ℓ a value that is "sufficiently large" so that the associated variable cannot have a non-zero value in any optimum solution. Notice, finally, that still other artifacts[2] are available so that, for instance, the selection $t_d = 0$ in Figure 2 reproduces a situation in which there is no increase in the time required to go from node 2 to node 8 via node 5.

1/ Other more compact representations are possible, but the one used here has been selected for its greater suggestive power.

2/ For a discussion of the use of various artifacts (and other devices) for synthesizing models see A. Charnes and W. W. Cooper "Elements of a Strategy for Making Models in Linear Programming" in Chapter 26 of Systems Engineering Handbook, R. Machol, et. al., eds. (New York: McGraw-Hill, Inc., 1965).

4. Multi-Copy Networks:

Rather than pursue such further possibilities for extensions to the problems of Figures 1 and 2, we now turn to the problem of synthesizing a "multi-copy network" for accommodating multiple origin-and-destination requirements, two-way traffic flow, etc., as encountered in the CATS study. For this purpose refer to Figure 3 below. Each of the rectangular boxes displayed there is supposed to contain an incidence array of $\pm$ 1 and 0 values, etc., in a format similar to Table 1. Here, however, there are m (e.g., m=100) such arrays, each with its own influxes and effluxes to indicate origin-and-destination requirements as well as suitably positioned zeros for nodes that are neither an origin nor a destination. See the right-hand column of rectangles in Figure 3.

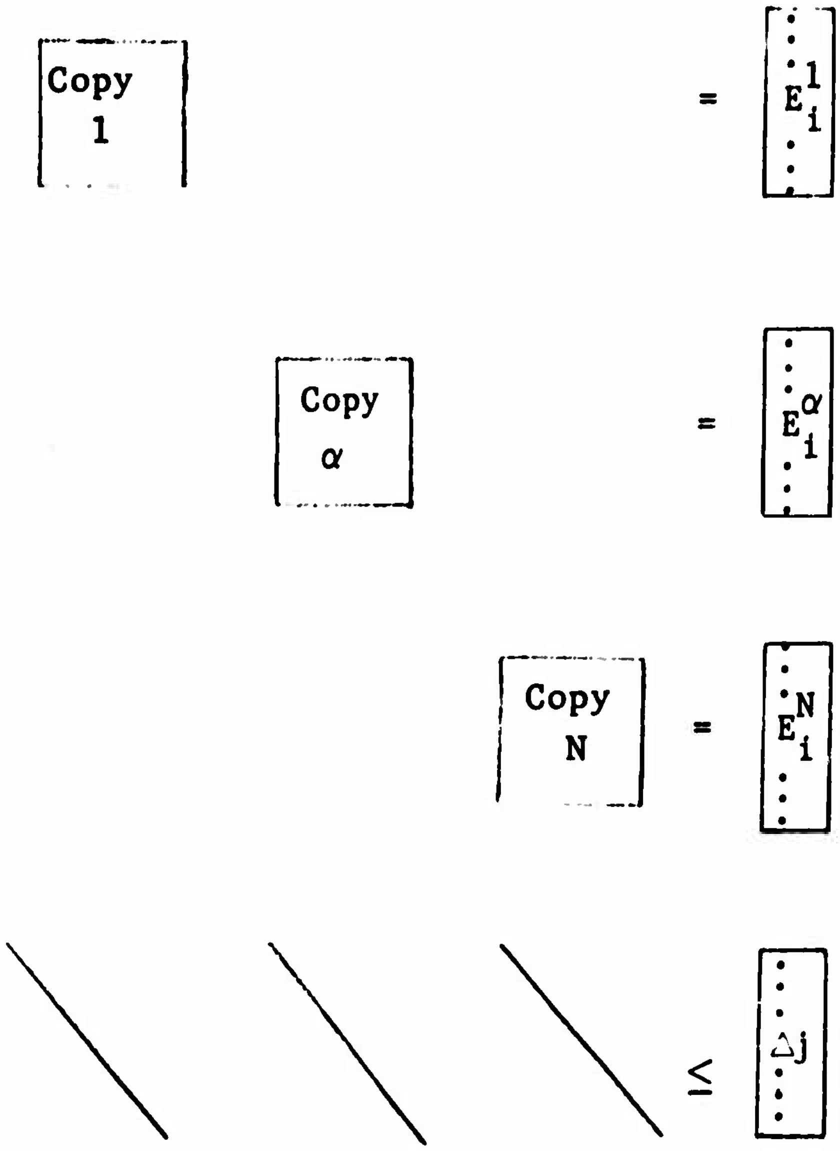


FIG. 3 Schema for a multicopy network

Next refer to the slashes shown at the bottom of this Figure and observe that each diagonal is positioned below a rectangular box. These lines schematically portray the multicopy extension of (5) which, it may be recalled, was the constraint used to accommodate part of the nonlinear

reaction via $q_4 \leq 4$. Here, however, more than a single copy is involved and since the applicable condition is responsive to total volume we might write, say,

(6) $$q_4^1 + \cdots + q_4^{\alpha} + \cdots + q_4^N \leq 4$$

to mean that the time to transit this link is the one provided for the applicable rectangle only so long as this constraint is satisfied. Thereafter, an increase in the transit time will be experienced in a manner that is wholly analogous to the one previously discussed for link 2-5 in Figure 1. Thus, for instance, when these q_4 values attain equality in (6) then the next applicable condition is

(7) $$q_5^1 + \cdots + q_5^{\alpha} + \cdots + q_5^N \leq \Delta ,$$

and so on.

Evidently various devices for compression can be utilized and attention is also invited to the examination of alternate modes of representation and special modes of solution via graphical as well as algebraic means. To make this somewhat more concrete, reference may be made to Figure 4, below, which contains data secured from traffic studies in a relatively small Indiana town.[1/] Consider, for instance, the link going from node (B) to node (A) which is designated as link 1 in this

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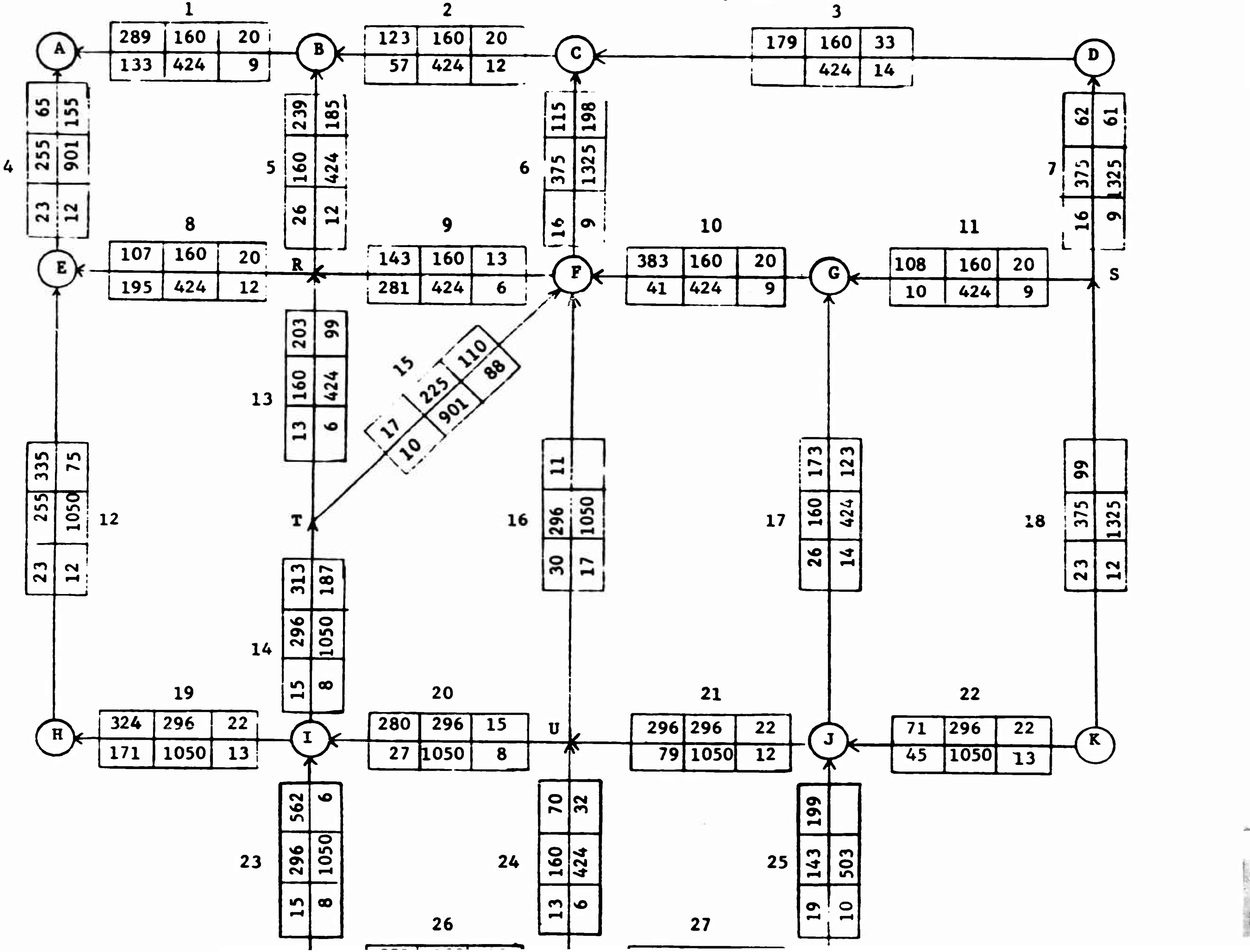
1/ Data supplied by J. Douglas Carroll, Jr., Elizabeth Gardner, Howard Bevis and Peter Caswell who, as members of the staff at CATS, collaborated with the authors in this research.

Figure. Notice that certain numerical data are portrayed there. Reading from right to left these data are to be interpreted so that only 9 minutes is required to traverse this link, provided the vehicular volume does not exceed 424 units. Up to 160 additional vehicles may also be assigned to this link but the transit times for these additional vehicles is then raised to 20 minutes. See the numerical entries in the four boxes on the right (nearest the tail) of link 1. Because only a two-step approximation is involved,[1/] no more than a total of 584 (= 424 + 160) vehicles can be assigned to this link for the indicated simulations.

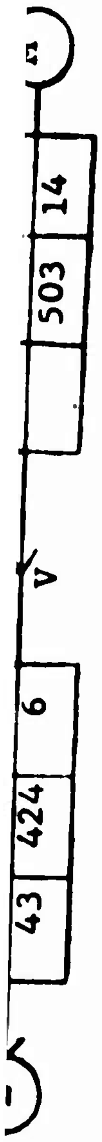
The solution that was finally achieved produced the vehicular flow volumes that are shown in the two boxes immediately adjacent to the arrowhead on link 1. These numerical values are meant to show that 289 vehicles are assigned for flow in the direction of the arrow--viz., from B to A --while 133 vehicles are assigned for flow in the opposite direction--*viz.*, from A to B --with a resulting total assignment of 422 (= 289 + 133) vehicles flowing in *both* directions on this link.

1/ Further increments can be added if desired, of course, or finer degrees of approximation can be used but the two-step ones shown in Figure 4 were deemed adequate for test checking the computational routines that were being developed during this phase of the research.

Figure 4. Characteristics of Arterial Network and Optimal Loads



The data shown on the other links of Figure 4 are interpreted in an analogous manner--*viz.*, the numbers in the boxes closest to the tail of an arrow show the transit times with the related initial and incremental vehicular volumes shown in the boxes positioned immediately alongside their times. Finally, the solution values required to fulfill the origin-destination requirements are shown immediately next to the arrowhead with the numer on top depicting traffic in the same direction as the arrow and the amount of cross-stream traffic (i.e., number of vehicles flowing in the opposite direction) shown at the bottom.



The data for these origin and destination requirements and the nodes with which they are associated may be conveniently presented in the format of Table 3. Here a negative number indicates an *origin* amount while a positive number indicates a *destination* requirement. In each case the node of origin or destination is shown in the stub. Nodes like R, S, T, U and V that have no numbers positioned alongside them are to be considered as neither origins nor destinations. *Vide* also Figure 4.

Each *origin* is now assigned a street network *copy* in the manner described for Figure 3. Thus, for instance, copy 1 is associated with node M where -575 shows the number of vehicles originating there. In this same "copy-1" column the various destinations for these 575 vehicles are also recorded. Thus, by reference to this column, it may be seen that 50 of these vehicles have node A as their destination, 35 are destined for B, zero for node C, and so on.

In a similar way the second column of Table 3 is associated with -170

at node K. Thus 170 vehicles have this node as origin whereas the nodes positioned in the stub opposite the positive numbers under the column labeled "copy 2" give the destination location and amounts. Blanks in any case indicate that the corresponding node is neither an origin nor a destination for the indicated copy.

Table 3
Origin and Destination Requirements

node \ comy	1	2	3	4	5	6	7	8	9	10	11
A	50	14	22	24	38	5	35	42	45	-209	
B	35	11	32	18	27	6	34	34	-233		
C			2	62	20	-54					
D	10	27	-215								
E	55	16	24	27	71	9	46	43	52	103	-184
F	60	19	29	31	45	10	84	54	86	57	121
G	40	12	30	20	26	7	-329				
H	170	7	12	22	-318						
I	55	18	27	29	64	9	46	120	50	49	63
J	55	14	31	23	27	8	84	-293			
K	45	-170									
L		32	6	-256							
M	-575										
R											
S											
T											
U											
V											

Negative sign denotes an influx.

Every column of Table 3 evidently sums (algebraically) to zero since, supposedly, every originating vehicle has some node as its destination.[1/] The row sums, on the other hand, show the net number of vehicles to be received or despatched at the node which is apparent in the stub. Thus at node A the net column sum is 275 - 209 = 66, which means that there should be a net inflow of 66 vehicles at this point in the system. To confirm that the solution shown on Figure 4 satisfies this net requirement at node A we can observe that (i) there is an inflow from nodes B and E to A in the amounts 289 + 65 = 354 whereas (ii) the total flowing from A to these nodes is 133 + 155 = 288 (see Figure 4), so that (iii) the net difference is 354 - 288 = 66, as required.

Of course something more than these totals may be wanted for purposes of redesigning the underlying schema. For instance, there may be some interest in identifying the sources (both origins and destinations) which occasion the flows on various branches.[2/] These, too, may be identified very easily by reference to the copies from which the total model was synthesized and the results of the computations may then be arranged as in Table 4.

1/ Including perhaps "phony nodes" introduced for the purpose of allowing vehicles to leave the system at some point and return to it at others.

2/ E.g., an interest of the indicated kind may arise from the need for considering the vehicle-related consequences of rezoning as well as urban-industrial development proposals.

Table 4

Branch Flows by Copies

Branch \ Copy	1	2	3	4	5	6	7	8	9	10	11
1	50	14	22	24	$\overline{27}$	5	35	42	97	$\overline{106}$	
2		25	54			11	33			$\overline{57}$	
3		25	154								
4					65				$\overline{52}$	$\overline{103}$	
5	85			42			36	76	$\overline{136}$	$\overline{49}$	
6			$\overline{98}$	62	20	$\overline{43}$	33			$\overline{57}$	
7	10	52	$\overline{61}$								
8		16	36		$\overline{74}$	9	46				$\overline{121}$
9		16	36		$\overline{74}$	9	82		$\overline{86}$		$\overline{121}$
10	49	35			$\overline{26}$	$\overline{13}$	245	54			
11	$\overline{10}$	47	61								
12	55		$\overline{12}$	27	210			43			$\overline{63}$
13	85			42				76	$\overline{50}$	$\overline{49}$	
14	85		$\overline{33}$	135	17	$\overline{9}$	$\overline{46}$	76	$\overline{50}$	$\overline{49}$	
15			$\overline{33}$	93	17	$\overline{9}$	$\overline{46}$				
16	11										
17	99		$\overline{31}$	20		$\overline{8}$	$\overline{84}$	54			
18		99									
19	225	7		49	$\overline{108}$			43			$\overline{63}$
20	16	25			$\overline{27}$			239			
21		57		$\overline{43}$	$\overline{27}$			239			
22	$\overline{45}$	71									
23	349		$\overline{6}$	213							
24	27	$\overline{32}$		43							
25	199										
26	349	32		$\overline{43}$							
27	376										

A bar denotes flow direction opposite to the Figure 4 branch arrow.

Here a barred figure represents the flow of traffic in the direction that is oppositely oriented to the arrow on the indicated branch. Thus the cross-stream flow of 133 vehicles that was previously noted for the solution values on branch 1 of Figure 4 are now seen to originate as follows: 27 of these vehicles flow cross-stream on this branch by virtue of the origin-destination requirements for copy 5 while the other 106 are occasioned by the requirements of copy 10. Cf. Table 4.

The unbarred numbers in Table 4 all refer to vehicular flow in the same direction as the arrow on the indicated branch. Thus, summing these values for branch 1 in Table 4 produces $50 + 14 + 22 + 24 + 5 + 35 + 42 + 97 = 289$, the same result that was recorded for this link in Figure 4. In each case the sources that occasion these flows on any branch may thus be identified, provided the following factors are born in mind: (i) each copy is composed from a battery of both origin and destination requirements and also (ii) the flows which emerge are occasioned by a "shaking down" process in which all of the copies and their associated origin-destination requirements are, in general, jointly considered. Thus any simulation and its controlling optimizations, if any, must be oriented with reference to these joint interaction possibilities.

5. Polyextremal Characterizations and Equilibrations:

It is now convenient to consider the types of optimization that might be used and perhaps the easiest entrance to this topic is via the game-theoretic constructs that were used in the course of this research. Suppose, therefore, that a player is associated with each origin who tries to locate routes that will minimize the total time required for his vehicles to reach their destinations.[1/] Note, however, that more than one player is involved in this game, and that each is trying to minimize his total time.[2/] Also the times available to each player are partly determined by the actions of the other players. Thus, these times are completely specified only when the actions of all of the players are known. Finally, it may also be noted that, in general, none of these many players may collude with any of the others in this "multi-person, non-zero-sum game." The single optimization (i.e., total time minimization) of Figure 1 must apparently then give way to a multiple optimization that allows for time alterations that may attend the choices of each of several players. This results in what we may refer to as a "polyextremal" problem or characterization.

These considerations give rise to a very natural situation for utilizing the ideas of "non-cooperative games" and their associated "Nash equilibrium"

1/ "Artificial" or "phony" origins may be employed when different types of vehicles (which congest) need to be considered. See, e.g., A. Charnes and W. W. Cooper, "Multicopy Traffic Network Models" in R. Herman, ed., Theory of Traffic Flow (Amsterdam, Elsevier Publishing Co., 1961).

2/ This interpretation was used to give analytical content to one of two rational (qualitative) principles of traffic flow over a network that had previously been enunciated by J. G. Wardrop as part of his work for the Road Research Board of the Department of Scientific and Industrial Research (Gr. Br.). See, e.g., A. Charnes and W. W. Cooper, op. cit. pp. 787 ff. In addition to this game theoretic characterization one can also employ other related concepts such as Pareto optimization, or, more generally, the ideas of functional efficiency as discussed in Chapter IX, loc. cit.

solutions. Attention toward these possibilities was invited in part because the existence of these "equilibrium" solutions for <u>any</u> such game had previously been established by J. F. Nash.[1/] This, however, was only an in-principle result and so it was therefore necessary to devise an algorithm which could be systematically and efficiently applied to secure these equilibrium solutions in the context of large-scale problems such as were involved in this case.

The evolution of such algorithms opened the way for possible subsequent applications of these game theoretic ideas to actual "management-sized" problems.[2/] They also provided routes for examining other pertinent issues including some which extended even beyond the case of the non-cooperative games that are comprehended in these Nash equilibria.

Consider, first, how the already available ideas associated with the stability of Nash solutions might be employed to check for solution properties such as sensitivity, etc. As already noted, a Nash equilibrium is related to the notion of a multiple optimization when no player enters into an explicit agreement to coordinate his choices with those of any other player.[3/] The attainment of such an equilibrium is evidenced by

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1/ <u>Vide</u>, J. F. Nash, "Non-Cooperative Games," <u>Ann. Math</u>. 54 (Sept., 1951).

2/ This case thus constitutes an exception to more usual situations wherein game theory has proved useful mainly for its conceptual power.

3/ When such agreements are allowed then the theory of cooperative games becomes pertinent.

the fact that no player can effect any alternate routings that will <u>reduce</u> the total time for his vehicles to reach their destinations.

There is, however, an important proviso that needs to be made in connection with this last statement. Given that the indicated equilibrium has been attained, it is first supposed that only <u>one</u> player at a time conducts such explorations. That is, the <u>other</u> players all adhere to their previously designated minimizing choices whenever any <u>one</u> player is conducting such explorations.

Suppose, however, that $k \geq 2$ players simultaneously effect such explorations. It need not be the case--indeed, for arbitrary k, it will generally <u>not</u> be the case--that two or more players acting simultaneously in the indicated manner will not be able to locate improved routings only because they were not able to do so separately. If, up to some number k, however, the already attained minimizations cannot be improved then the result is called a "k-stable variant" of the Nash equilibrium that was previously achieved.

Such k-stable analyses were deemed desirable and hence methods for conducting them also had to be devised. Thus, even though there is no real opportunity for these "fictitious" players to deliberately coordinate their explorations, such additional analyses could provide insight into the stability of possible solutions as a further aid to understanding and also as a possible way of effecting further choices between the different Nash equilibria that might emerge.

Beyond the chance possibilities of simultaneously executed explorations by 2 or more players, one might also note the possible utilization of ideas from the theory of cooperative games. Here something more than coordination by chance may be admitted. Indeed the players in a cooperative game may explicitly agree to coordinate their choices as well as the timing of their explorations. Such an agreement between two or more players is then called a (cooperative) "coalition."

To make these ideas more concrete it may be supposed that the players associated with copies 1 and 2 enter into such a coalition. See Table 3. This new player "One plus Two" then seeks to minimize the time for the resulting total of 745 (= 575 + 140) vehicles that are under his control. Evidently the minimum he can attain will be at least as small as the total of the two minima that the players for copies 1 and 2 can separately achieve.

The allowance of such cooperative coalitions leads into rather deep problems such as the following: (i) how should the resulting minimum be distributed between the participants in such a coalition and (ii) what (possibly best) coalitions will be formed? The kinds of uses and interpretations envisioned here, however, need not necessarily deal with questions such as these in all detail and generality. They may instead be addressed to such issues as consolidation and aggregation. Notice, for instance, that some prior aggregation may be involved in the choice of these fictitious players. Thus the analyst may wish to test these already effected

consolidations by deconsolidating some of them. For instance, he may wish to deconsolidate the previous node M into two new ones M_1 and M_2, with a designated number of the 575 vehicles (at M) assigned to each. Conversely, he may wish to study the consequences of combining the origin and destination requirements for copies 1 and 2, and so on.

For such purposes an analyst may need to consider only such issues as convenience and testing--e.g., testing already available results when making his coalition (i.e. consolidation or deconsolidation) choices. Also in effecting his distributions he may proceed in a variety of ways --e.g., by various rules of priority, and so on.

Other uses of existing game theoretic constructs are also possible. Although many of these possibilities for polyextremal (i.e., multiple optimization) characterizations and deployment were considered (and used), they did not meet all of the requirements of the instant study. In particular, the possibilities for subsequent use in redesign as well as the need for increased computational efficiency made it desirable to consider whether additional research might provide results that could yield access to the related field of mathematical (possibly nonlinear) programming. It was known, for instance, that an exact relation had already been established between two-person zero-sum games and ordinary linear programming. Rather surprisingly the research that was conducted showed how formulations could also be developed, however, which made it

possible (a) to replace the above polyextremal approach with an alternate that could proceed within the framework of a single (overall) optimization and (b) to devise special computational routines which were even more efficient those those of ordinary linear programming when applied to the multi-copy networks such as the one that is depicted generically in Figure 3. In fact, the solutions discussed in the previous sections were all obtained via these results.[1/]

6. Evaluations and Extensions:

Turning now to Table 5, below, we may illustrate some of the byproducts that immediately became available from the linear programming formulations and solution routines that were ultimately devised.[2/] First refer to the values listed at the bottom of Table 5 as

$$\Psi_5 = 2,\ \Psi_9 = 1,\ \Psi_{10} = 4,\ \Psi_{17} = 0,\ \Psi_{26} = 0.$$

These refer to the levels at which the nonlinear time alterations are experienced on the links designated by the subscripts. For instance Ψ_5 refers to link 5 in Figure 4 where the transit time is altered from 12 minutes to 26 minutes when more than 424 vehicles are utilizing it. Suppose now that an improvement (e.g., street widening) is to be considered which will raise this to 425 units instead of the indicated 424 units.

1/ Acknowledgment should be made to Dr. A. Ben-Israel for his collaborative efforts in this part of the research while he was a student at Northwestern University. Subsequently A. Charnes and C. E. Lemke extended these results to so-called multi-page networks in which the separate copies of Figure 3 are permitted to differ from one another in their structure.

2/ It should first be emphasized, however, that no extra computations are involved since these data are truly byproducts of the solution routines that were developed.

The value $\Psi_5 = 2$ means that a 2 minute reduction in total travel time will then be experienced over <u>all</u> copies of the network from the new optimal loadings that then become possible. Moreover, an additional 2 minute reduction in overall travel time will also be experienced as the level is further raised to 426 vehicles, and so on.[1/] Indeed, combinations of these Ψ values such as $\Psi_5 + \Psi_{10} = 2 + 4 = 6$ may also be utilized as guides for selecting "best patterns" of link alterations to secure overall network performance. The resulting benefit measures can then be compared to their costs, if wanted, although for some purposes it may be better to enter these costs into a model that is specifically designed for this purpose.--e.g., when a stipulated budget is to be utilized optimally. [2/]

These Ψ values are applicable across <u>all</u> copies of the network and hence have been listed at the bottom of Table 5. Other byproduct data are also produced that can be identified with <u>each</u> of the copies. Some of these data are listed in the body of Table 5 and arranged for easy reference to the individual copies (and nodes). To illustrate some of their possible uses we may first refer to copy 1 in Table 3 in order to observe that M there has node B as one of its destinations. Then we may refer to Table 4 and observe that branch 2 is not utilized by M even though this link (from C to B) forms part of a possible route from M to B.[3/]

1/ The byproduct data also makes it apparent how far this process may be continued before this Ψ_5 values is changed. See, e.g., A. Charnes and W. W. Cooper, <u>op</u>. <u>cit</u>.

2/ Or one may proceed via devices analogous to those used in PERT/Cost network studies. See also T. M. Ridley, "An Investment Policy to Reduce the Travel Time in a Network" Operations Research Center, University of California, Berkeley, Dec., 1965.

3/ See Figure 4.

The transit time from C to B on link 2 is 12 minutes and so a question naturally arises as to the magnitude by which this time must be reduced in order to make its use attractive for M. An answer can be secured from Table 5 by subtracting the copy 1 values opposite B and C to secure[1/]

(8) $$\phi_B^1 - \phi_C^1 = 56 - 46 = 10.$$

This means that the present transit time of 12 minutes would need to be reduced to 10 minutes before M would begin to consider it for use as part of its optimal routings.

Next observe the value $\phi_A^1 = 65$. As may be observed from Table 4 and Figure 4 the 50 vehicles that M routes to A proceed via the branches 27, 26, 23, 14, 13, 5 and 1. Because branch 5 with $\Psi_5 = 2$ is utilized, we have

(9) $$\phi_A^1 - \Psi_5 = 65 - 2 = 63$$

as the total time for each vehicle so that this part of M's optimal loading yields a total of 63 x 50 = 3150 vehicle minutes.

The value $\phi_A^1 = 65$ also provides an "opportunity cost" evaluation in the following sense. Suppose that link number 5 were eliminated from

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1/ These symbols should be read so that ϕ_B^1 means "the value, ϕ, for node B, on copy 1."

Table 5

Node and Capacity Evaluators

cap / node	1	2	3	4	5	6	7	8	9	10	11
A	65	56	35	45	24	21	43	57	9	0	12
B	56	47	26	36	33	12	34	48	0	9	21
C	46	35	14	35	40	0	22	36	12	21	28
D	44	21	0	56	54	14	18	32	26	35	42
E	53	53	42	33	12	28	32	46	21	12	0
F	37	34	23	26	31	9	13	27	21	30	19
G	24	21	18	38	44	22	0	14	34	43	32
H	41	46	54	21	0	40	44	34	33	24	12
I	28	33	41	8	13	27	31	20	28	37	25
J	10	13	32	24	33	36	14	0	48	57	45
K	23	0	21	37	46	35	21	13	47	56	53
L	20	37	49	0	21	35	38	24	36	45	33
M	0	23	42	20	41	46	24	10	56	65	53
R	42	41	30	22	24	16	20	34	14	23	12
S	35	12	9	47	53	23	9	23	35	44	41
T	36	41	33	16	21	19	23	28	20	29	18
U	20	25	40	12	21	26	26	12	36	45	33
V	14	31	46	6	27	32	32	18	42	51	39

$\Psi_5 = 2, \quad \Psi_9 = 1, \quad \Psi_{10} = 4, \quad \Psi_{17} = 0, \quad \Psi_{26} = 0$

the network. As may then be confirmed from Figure 4, the best routing from M to A would then be via links 27, 26, 23, 19, 12 and 4 with a time requirement equal to the value of ϕ_A^1--viz.,

$14 + 6 + 8 + 13 + 12 + 12 = 65$ minutes.

As a final illustration of the further evaluation possibilities we might consider the consequences of reducing the load moving from M to B. Thus suppose, for instance, that the number of vehicles with B as their destination is reduced from 35 to 34 while, simultaneously, the number originating at M is reduced from 575 to 574.[1/] Because link 5 is utilized when proceeding from M to B we adjust the value $\phi_B^1 = 56$ to $\phi_B^1 - \Psi_5 = 56 - 2 = 54$. Adding the value $\phi_M^1 = 0$[2/] to this result produces no alteration and hence we can assert that each such reduction in M's load will produce a reduction of 54 vehicle minutes for the system.

7. Some Further Directions for Research:

As will be evident from Table 5, better reductions in the system's travel time can be effected by reference to reductions other than the one we have just used for illustration. Moreover, transfers of location

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1/ This *pari passu* reduction is required in order to maintain the net column balance at zero for copy 1 in Table 3.

2/ The computational routines were arranged so that these analyses would be facilitated by assigning a ϕ value of zero to all origin nodes. Cf. Tables 3 and 5.

for these vehicle loads will also produce reductions (or increases) in total travel time and these, too, may be used for rezoning, urban-industrial development projects and other such studies. Finally, there are still further possibilities that can be obtained from using combinations of Tables 3, 4 and 5 in a variety of ways.

We have previously indicated some of the ways in which the above model may be elaborated (e.g., by reference to chance-constrained programming) and among these we may include the kinds of vector optimizations that are are available via the ideas of "functional efficiency" and related concepts such as "goal programming," etc.[1/] Such ideas may be useful when designs are wanted that will not produce an undue worsening of the travel times that might otherwise occur in some of the copies. when a redesign is undertaken.

Having noted the background of this research in computer aided simulation studies it is perhaps well to conclude by noting some of the added possibilities for research in these directions. The network ideas elaborated here would appear to suggest the desirability of exploring analogue as well as digital devices. Something might be done to facilitate such system studies if a suitable analogue device were specifically developed for this purpose. As the preceding discussion

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1/ See Chapter IX and Appendix B in A. Charnes and W. W. Cooper, *op. cit.*

is meant to suggest, however, an ideal device of this kind should service the need for evaluating possible alterations in the system as well as supplying a convenient way to simulate the behavior of traffic on an already existing network.

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13. ABSTRACT

A sequence of linear programming models of network type are here used to illustrate how the optimizations of linear programming may be used to provide guidance and control (a) for simulating complex nonlinear systems and (b) for evaluating possible alterations in system designs. This is first illustrated by an example involving only a single extremization (=optimization). Subsequently this is extended to a polyextremization which utilizes certain concepts from the theory of n-person non-zero sum games. The latter is then replaced by a model which again utilizes only a single extremal principle which is related to linear programming by means of what are called multi-copy network models. This is used to accommodate multiple origin-to-destination requirements in which two-way flow on the links is possible. Possible extensions and use of these ideas are examined, including ways in which zoning and traffic studies might be combined for joint treatment. Routes for further research are also suggested.

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